Fast and accurate simulation of periodic flow conditions in piping systems

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ABSTRACT
This paper presents an accurate and efficient method for simulating oscillating flow conditions in piping systems. Solving the pipe flow equations in the frequency domain is much faster than solving them in the time domain, but this requires the non-linear friction term to be replaced by a linear approximation based on the steady state flow rate. Therefore, unrealistically high oscillations can be predicted in piping sections where the steady state flow rate nears zero. The authors of this paper propose two modifications that make the solution method almost as accurate as the method of characteristics in realistic piping systems.

1 INTRODUCTION

Oscillating and pulsating flow conditions in piping systems occur when external energy sources or internal excitation mechanisms cause periodic oscillations in the flow rate and the pressure. External energy sources are typically positive displacement machines such as screw compressors and reciprocating compressors and pumps. A common internal excitation mechanism is the periodic shedding of vortices at junctions and control valves.

Oscillating flow conditions can become problematic if they lead to acoustic resonance (1). This happens when pressure waves are reflected at geometrical discontinuities and interfere with themselves in such a way that superimposed wave crests amplify each other at specific locations in the piping system. The resulting oscillations in the pressure and flow rate may then become so large that they lead to significant noise and vibration levels. In severe cases acoustic resonance may lead to damage in piping and equipment due to fatigue and pressure limits being exceeded. Any resulting leaks may pose risks to people and the environment.

Fortunately, acoustic resonance can be avoided by (small) changes in the geometry of the piping system, by installing pulsation dampening devices and by modifying pipe supports. With the help of computer simulation tools engineers can assess the effectiveness of different resonance-countering measures before any changes to the piping systems are implemented. Those same tools can be used to design new systems in which the flow and pressure oscillations remain below acceptable levels, avoiding the need for costly modifications after the system has been deployed for operation.

There are essentially two numerical methods for simulating oscillating flow conditions in piping systems: the time-domain method and the frequency-domain method. The latter is commonly referred to as the harmonic solution method for a reason that will become apparent in a moment.

The time-domain method involves the use of a transient solution algorithm – typically the method of characteristics (2) – for solving the time-dependent and non-linear equations describing the flow of a fluid through piping systems. A discrete Fourier transform is applied afterwards to obtain a breakdown of the flow and pressure oscillations in the frequency domain. The advantage of the time-domain method is that any transient flow solver can be used to simulate oscillating flow conditions. Another advantage is that the non-linear friction term in the flow equations can be properly accounted for. The major disadvantage of this approach is that simulations can be time consuming, especially for non-trivial piping systems.

The harmonic solution method involves a transformation of the time-dependent flow equations to the frequency domain, resulting in a system of complex, harmonic equations (3). The solution of these equations directly yields a frequency breakdown of the flow and pressure oscillations; there is no need to apply a discrete Fourier transform afterwards. The main advantage of the harmonic solution method is its computational speed; it tends to be one or two orders of magnitude faster than the time-domain method. A secondary advantage is that the geometry of the piping system is accounted for in an exact way. This is, in general, not possible when using the method of characteristics. The major drawback of the harmonic solution method, at least in its original form, is that it uses the steady state (or average) flow rate to obtain a linear approximation of the non-linear friction term. This can result in inaccurate predictions of flow and pressure oscillation levels, especially in piping sections where the steady state flow rate tends to zero.

The accuracy of the harmonic solution method can be improved by adopting a fully non-linear solution scheme so that the non-linear friction term is properly taken into account (4, 5). Unfortunately, this approach significantly increases both the computation time and the complexity of the solution algorithm, especially when including models for equipment such as valves, orifice plates and pulsation dampeners.

Fortunately, it turns out that a significant gain in accuracy can already be achieved by using a weakly non-linear approximation of the friction term. This approximation replaces the steady state flow rate by an effective friction flow rate in such a way that the energy losses in the frequency domain equal the energy losses in the time domain. Although the resulting algorithm is nonlinear, its core is essentially the same as the original, linear solution algorithm so that its implementation is relatively straightforward.

The next section continues this paper with a description of the original, linear harmonic solution method. Section 3 then explains how this method is modified to obtain a more accurate approximation of the friction term. Next, Section 4 assesses the accuracy and performance of the modified harmonic solution method by comparing it with the original, harmonic solution method and the time-domain solution method (method of characteristics). Finally, Section 5 concludes this paper.

2 THE LINEAR HARMONIC SOLUTION METHOD

The flow of a single-phase fluid through a pipe is described by the following momentum and continuity equations:

$$\begin{aligned} \frac{\partial p}{\partial x} + \frac{1}{A}\frac{\partial \phi}{\partial t} - \rho g \, \sin\alpha + R|\phi|\phi &= 0 \\ \frac{\partial p}{\partial t} + \frac{c^2}{A}\frac{\partial \phi}{\partial x} &= 0 \end{aligned} \tag{1}$$

where p is the pressure, ϕ is the mass flow rate, R is a compound friction coefficient $[\mathrm{kg}^{-1}\mathrm{m}^{-2}]$, ρ is the fluid density, c is the wave speed, A is the cross-sectional pipe area, g is the gravitational acceleration, and α is the inclination angle of the pipe. Note that these equations assume that the pressure and flow velocity are constant over any pipe cross section, and that the flow velocity is significantly lower than the wave speed.

The friction coefficient R is assumed to be constant. This is an approximation because in reality the friction coefficient will vary in time. Common unsteady friction models, such as the one proposed by Zielke (8), involve a convolution integral that is cumbersome to evaluate in the time domain. In the frequency domain, however, the convolution integral becomes a straightforward multiplication. This advantage of the frequency-domain method has not been exploited in the context of this paper, mainly because unsteady friction was not supported by the time-domain solver that was used to calculate the reference results.

The steady state solution of the flow equations plays an important role in the solution method presented below. The steady state solution satisfies the flow equations when the time derivatives equal zero and all time-dependent flow boundary conditions have been replaced by constant flow boundary conditions equal to their time average. Thus, the steady state solution is composed of the mean flow rate and pressure distribution. When one would compare the fluid flow to an electric current, then the steady state solution could be viewed as the DC component of an AC signal.

Transformation to the frequency domain

When a piping system is subjected to a periodic excitation mechanism with a base period T, then the flow rate and pressure within the system will also be periodic with the same base period. This means that the mass flow rate and pressure can be expressed as the following Fourier series:

$$\begin{aligned} p(x,t) &= P_0(x) + \sum_{k\neq 0} P_k(x)e^{-ik\omega t} = P_0(x) + p'(x,t) \\ \phi(x,t) &= \Phi_0(x) + \sum_{k\neq 0} \Phi_k(x)e^{-ik\omega t} = \Phi_0(x) + \phi'(x,t) \end{aligned} \tag{2}$$

Where

$$\omega = \frac{2\pi}{T}$$

Here P_0 and p' denote the steady state pressure and the complex-valued pressure oscillations, respectively. The steady state mass flow rate and flow oscillations are denoted in a similar way. The momentum and continuity equation can therefore be written as:

$$\begin{aligned} \frac{dP_0}{dx} - \rho g \sin\alpha + \frac{\partial p'}{\partial x} + \frac{1}{A}\frac{\partial \phi'}{\partial t} + R|\Phi_0 + \phi'|(\Phi_0 + \phi') &= 0 \\ \frac{c^2}{A}\frac{d\Phi_0}{dx} + \frac{\partial p'}{\partial t} + \frac{c^2}{A}\frac{\partial \phi'}{\partial x} &= 0 \end{aligned} \tag{3}$$

To continue from here, the friction term is replaced by a linear approximation as follows:

$$R|\Phi_0 + \phi'|(\Phi_0 + \phi') \approx R|\Phi_0|\Phi_0 + 2R|\Phi_0|\phi' \tag{4}$$

This means that the momentum equation becomes:

$$\frac{dP_0}{dx} - \rho g \sin\alpha + R|\Phi_0|\Phi_0 + \frac{\partial p'}{\partial x} + \frac{1}{A}\frac{\partial \phi'}{\partial t} + 2R|\Phi_0|\phi' = 0 \tag{5}$$

The steady state pressure and flow rate satisfy the following equations:

$$\frac{dP_0}{dx} - \rho g \sin\alpha + R|\Phi_0|\Phi_0 \quad = 0 \qquad\qquad \frac{d\Phi_0}{dx} \quad = 0 \tag{6}$$

This means that the pressure and flow rate oscillations are given by:

$$\begin{aligned} \frac{\partial p'}{\partial x} + \frac{1}{A}\frac{\partial \phi'}{\partial t} + 2R|\Phi_0|\phi' &= 0 \\ \frac{c^2}{A}\frac{\partial \phi'}{\partial x} + \frac{\partial p'}{\partial t} &= 0 \end{aligned} \tag{7}$$

Expansion of the Fourier series then yields:

$$\begin{aligned} \sum_{k\neq 0}\left(\frac{dP_k}{dx} - ik\omega\frac{1}{A}\Phi_k + 2R|\Phi_0|\Phi_k\right)e^{-ik\omega t} &= 0 \\ \sum_{k\neq 0}\left(-ik\omega P_k + \frac{c^2}{A}\frac{d\Phi_k}{dx}\right)e^{-ik\omega t} &= 0 \end{aligned} \tag{8}$$

These two equations can only be zero for every time t if for all $k \neq 0$ the following equations are true:

$$\begin{aligned} \frac{dP_k}{dx} - \frac{ik\omega}{A}\Phi_k + 2R|\Phi_0|\Phi_k &= 0 \\ -ik\omega P_k + \frac{c^2}{A}\frac{d\Phi_k}{dx} &= 0 \end{aligned} \tag{9}$$

Note that the above equations only need to be solved for positive values of k because $k$because $P_k = \bar{P}_{-k}$ and $\Phi_k = \bar{\Phi}_{-k}$ where $\bar{C}$ denotes the complex conjugate of C.

Linear harmonic solution procedure

Following the procedure described by Vitkovsky et al. (3) and Silva et al. (6) the above equations can be transformed into a linear, complex system of harmonic equations with the following form:

$$\boldsymbol{H}_k \boldsymbol{x}_k = \boldsymbol{y}_k \tag{10}$$

in which $\boldsymbol{H}_k$ is a square, complex matrix that depends on the system geometry, fluid properties and friction coefficients; $\boldsymbol{x}_k$ is a complex vector composed of the harmonic pressure oscillations P_k (amplitude and phase angle) at the nodes joining the pipe segments and other flow elements; and $\boldsymbol{y}_k$ is a complex vector containing the harmonic mass flow boundary conditions at the nodes. The solution of this system therefore yields the pressure amplitudes and phase angles at all nodes for the *k-th* harmonic frequency. From these the mass flow rate amplitudes and phase angles can be computed. Solving the system for all harmonic frequencies of interest (or values of k) therefore yields a complete breakdown of the pressure and flow rate in the frequency domain. As the matrix $\boldsymbol{H}_k$ tends to be very sparse, solving the system of equations only takes a fraction of the time that would be required to calculate the harmonic pressure and flow rate oscillations by means of the time-domain solution method.

Low accuracy for dead-end branches

Unfortunately, the linear approximation of the friction term introduces two significant inaccuracies. The first involves the way that energy losses due to friction scale in a linear way with the steady state mass flow rate. Thus, there is no energy loss when the steady state mass flow rate is zero. Consequentially, when resonance modes are excited in piping sections in which the steady state mass flow rate is (nearly) zero, then unrealistically high pressure and flow oscillations will be predicted by the linear harmonic solution method. In reality, the quadratic relation between the mass flow rate and the friction loss will lead to a dissipation of energy and dampening of the pressure and flow oscillations.

The second inaccuracy is related to the way that the pressure and flow oscillations may affect the steady state (or average) pressure and flow rate distribution. That is, (large) flow oscillations may locally change the effective, average friction and therefore increase the average pressure loss. This is a phenomenon that cannot be captured when using a linear approximation of the friction term.

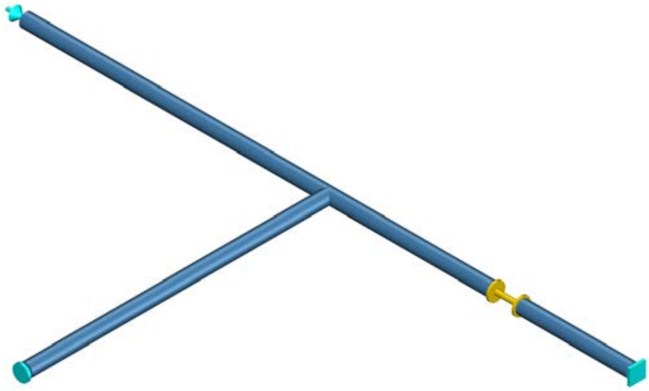


Figure 1: simple piping system consisting of a header pipe with an orifice plate and a dead-end side branch. The arrow at one end of the header pipe indicates a sinusoidal flow boundary condition. A fixed pressure boundary condition is applied at the opposite end.

Both types of inaccuracy can be illustrated clearly by considering the almost trivial piping system shown in Figure 1. The system consists of a header pipe with an orifice plate and a dead-end side branch halfway the header; see Table 1 for the relevant model parameters. At one end of the header pipe a pure harmonic (sinusoidal) flow rate is imposed. The oscillation frequency of this boundary condition is varied from 5*Hz* to 50*Hz*. At the other end of the header pipe a fixed pressure boundary condition is imposed. Because the steady state flow rate in the side branch equals zero, the pressure amplitude at the end of the branch as calculated with the linear harmonic solution method deviates vastly from the amplitude as calculated with the time-domain method; see Figure 2. The two methods also yield significantly different results when considering the average inlet pressure at the point where the flow boundary condition is applied; see Figure 3.

Table 1: The relevant parameters describing the dead-end side branch model.

Inner diameter	0.1 *m*
Header length	20 *m*
Branch length	10 *m*
Wave speed	1352 *m/s*

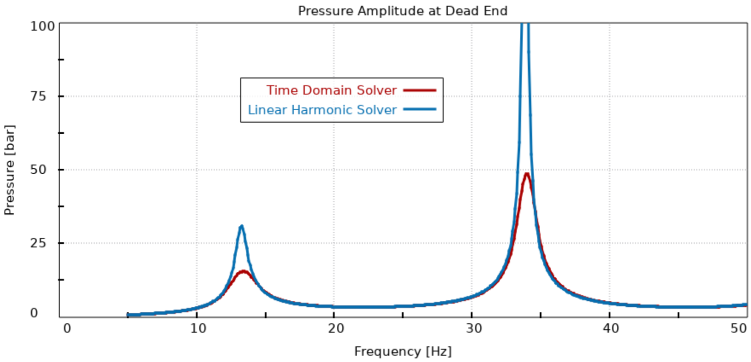


Figure 2: The pressure amplitude associated with the first harmonic component at the end of the branch as function of oscillation frequency of the applied flow boundary condition. The pressure amplitude has been calculated both with a time-domain solver (method of characteristics) and the linear harmonic solver. Resonance occurs at about 13*Hz* and 34*Hz*.

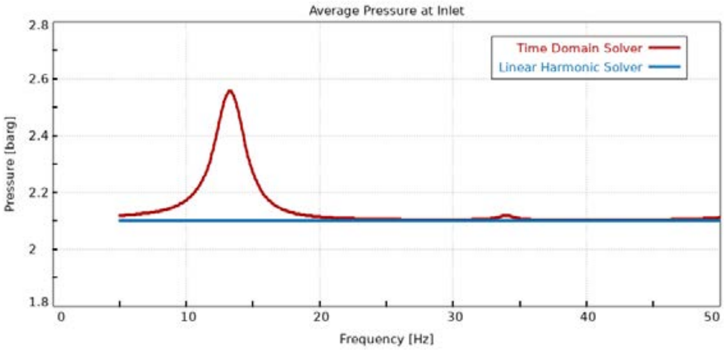


Figure 3: The average pressure at the inlet (where the flow boundary condition is applied) as function of the flow oscillation frequency. When using the linear harmonic solution method, the average flow rate equals the steady state flow rate that does not depend on the frequency.

3 THE MODIFIED HARMONIC SOLUTION METHOD

To improve the accuracy of the harmonic solution method, the authors propose two modifications that address the two types of inaccuracy caused by the linear approximation of the friction term.

Effective friction mass flow rate

The first modification involves replacing the linear approximation of the friction term (Equation 1) by an alternative form:

$$R|\Phi_0 + \phi'|(\Phi_0 + \phi') \approx R|\Phi_F|\Phi_0 + 2R|\Phi_F|\phi' \quad (11)$$

with $\Phi_F = \Phi_F(x)$ the effective friction mass flow rate at location *x* within a pipe section. It is determined by requiring that the original, non-linear friction term – as used in the time-domain method – leads to the same local energy dissipation as the above linear approximation of the friction term – as used in the harmonic solution method – within one time period. This equality must hold for any location *x* within any pipe section. The energy dissipation due to friction is given by:

$$\Delta E(x) = \int_0^T \left|\frac{1}{\rho}\Delta P_F(x,t)\phi(x,t)\right| dt \quad (12)$$

where ΔP_F is the pressure loss due to the friction. When using the non-linear friction term, the pressure loss is equal to:

$$\Delta P_F(x,t) = R|\phi(x,t)|\phi(x,t) \quad (13)$$

Therefore, the non-linear energy dissipation is given by:

$$\Delta E_{NL}(x) = \frac{R}{\rho}\int_0^T |\phi^3(x,t)|\, dt \quad (14)$$

When using the linear approximation of the friction term, the pressure loss is given by:

$$\Delta P_F(x,t) = R|\Phi_F(x)|\Phi_0(x) + 2R|\Phi_F(x)|\phi'(x,t) \quad (15)$$

This leads to the following expression for the linear energy dissipation:

$$\Delta E_L(x) = \frac{R}{\rho}|\Phi_F(x)|\int_0^T \left|\big(\Phi_0(x) + 2\phi'(x,t)\big)\phi(x,t)\right| dt \quad (15)$$

By setting $E_{NL}(x) = E_L(x)$ the following expression is obtained for the effective friction flow rate:

$$|\Phi_F(x)| = \frac{\int_0^T |\phi^3(x,t)|\, dt}{\int_0^T \left|\big(\Phi_0(x) + 2\phi'(x,t)\big)\phi(x,t)\right| dt} \quad (16)$$

Note that if the flow oscillations are much smaller than the steady state flow rate, then the effective friction flow rate tends towards the steady state flow rate.

The real mass flow rate $\phi(x,t)$ and the mass flow rate oscillations $\phi'(x,t)$ are not known, but they can be approximated by summing a finite number of harmonic components as follows:

$$\phi'(x,t) \approx \sum_{k=-n,k\neq 0}^{n} \Phi_k(x)e^{-ik\omega t} \quad (17)$$

In practice, only a few (ten to twenty, say) harmonic components need to be calculated to obtain a sufficiently accurate approximation of the mass flow rate (and its oscillations). The calculation procedure is explained later in this section.

Note that the effective friction mass flow rate Φ_F is used only when calculating the harmonic pressure and flow rate oscillations. It is not used when calculating the steady state solution. Another mechanism is used to adjust the friction losses when calculating the steady state solution; see below.

Steady state friction correction

The second modification to the linear harmonic solution method involves replacing the friction coefficient R by an effective friction coefficient R_F when solving the steady state flow equations. The goal here is to obtain a steady state solution that closer matches the time-average solution obtained with the time-domain method. This is achieved by requiring that the steady state pressure loss equals the average time-dependent pressure loss within one time period. This means that:

$$R_F(x)|\Phi_0(x)|\Phi_0(x) = \frac{1}{T}\int_0^T R(x)|\phi(x,t)|\phi(x,t)dt \quad (18)$$

The effective friction factor is therefore given by:

$$R_F(x) = \frac{\int_0^T R(x)|\phi(x,t)|\phi(x,t)dt}{T|\Phi_0(x)|\Phi_0(x)} \quad (19)$$

Again, the real, time-dependent mass flow rate is not known, but it can be approximated by combining a finite number of harmonic flow rate components; see Equation 17.

Note that the effective friction coefficient R_F is used only when calculating the steady state solution (Equation 6 with R_F instead of R). The regular friction coefficient R is used when calculating the harmonic pressure and flow rate oscillations (Equation 9).

The above equations are only valid for regular pipe segments, but they can easily be adapted for fittings like orifice plates and valves. This essentially involves replacing the friction factor by a discharge coefficient.

Non-linear solution procedure

The use of the effective friction mass flow rate and the effective friction coefficient turns the original, linear harmonic equations into a system of non-linear equations. That is, both the effective friction flow rate and the effective friction coefficient depend on the calculated harmonic mass flow rate components, which, in turn, depend on the effective friction mass flow rate and the effective friction coefficient. Fortunately, the non-linear dependency is rather weak which makes it possible to use a simple point iteration method that proceeds as follows:

- Set the effective steady state friction coefficient R_F to the regular friction coefficient R and determine the steady state solution. Then set the effective friction mass flow rate Φ_F equal to the steady state flow rate.
- Execute the linear harmonic solution method – using the current effective friction flow rate and steady state solution – to calculate the harmonic flow rate components.
- Calculate the new effective steady state friction coefficient R_F and update the steady state solution if necessary.
- Calculate the new effective mass flow rate Φ_F.
- Check the relative change in the effective friction mass flow rate; if it is small enough (less than 10^{-4}), then stop; otherwise execute the next iteration, starting at Step 2.

In practice the number of iterations that need to be executed tends to be small so that the overall algorithm is still much faster than the time-domain method. Actual performance numbers will be presented in Section 4.

Improved accuracy for dead-end branches

By using the effective friction flow rate and the effective friction coefficient, the harmonic solution method much better accounts for the non-linear friction term, especially when the steady state flow rate is small. Indeed, when considering the previous piping system with a single dead-end side branch (Figure 1), then the results obtained with the modified harmonic solution method are very similar to the results obtained with the time-domain method; see Figure 4 and Figure 5.

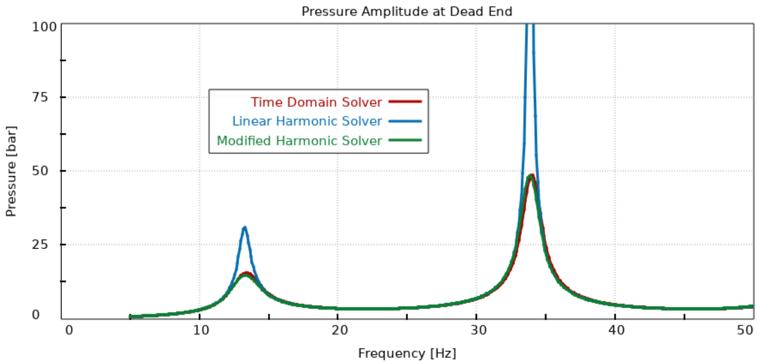


Figure 4: The pressure amplitude associated with the first harmonic component at the end of the branch as function of oscillation frequency of the applied flow boundary condition. The modified harmonic solver yields results that are very similar to the time domain solver (method of characteristics).

The modified harmonic solution method converges in three iterations for most of the frequency range that is displayed in the two figures. Close to the resonance frequencies the solution method requires more iterations, but never more than 18.

Coupling between harmonic components

There is one phenomenon that can be captured by the time-domain solution method but not by the harmonic solution method, even when using of the effective friction mass flow and the effective friction coefficient. This phenomenon involves a direct coupling between different harmonic components due to the non-linear nature of the friction term. That is, the amplitude of one harmonic component may directly affect the amplitude of other harmonic components. When using the modified harmonic solution method, there is an indirect coupling between the harmonic components through the effective friction mass flow rate, but this mechanism affects all harmonic components in the same way. In reality, each component may have a different effect on any other (higher-order) component.

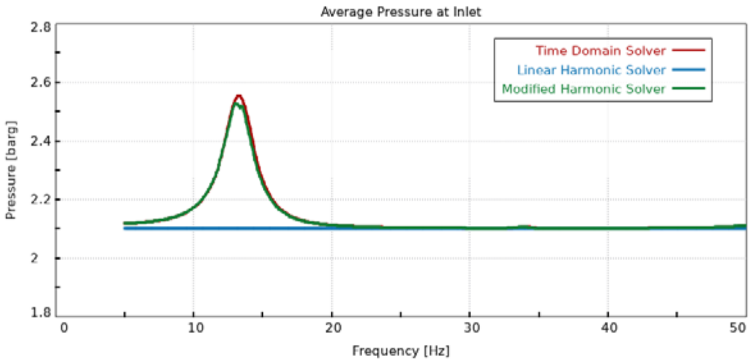


Figure 5: The average pressure at the inlet (where the flow boundary condition is applied) as function of the flow oscillation frequency. By using the effective friction coefficient, the modified harmonic solution method yields results that are very similar to the results obtained with the time-domain method.

This phenomenon becomes apparent when inspecting the pressure oscillations associated with the second harmonic in the example piping system. Because the flow boundary condition is a perfect sinusoidal function it is composed of only a single harmonic component. The modified harmonic solution method therefore calculates non-zero results only for the first harmonic component; the results associated with the second and higher-order harmonic components are identical to zero. The time-domain solution method, on the other hand, does calculate non-zero results for the higher-order harmonics; see the graph in Figure 6. This graph shows the pressure amplitude of the second harmonic component at the end of the dead-end side branch. Note that the second harmonic pressure amplitude is about two orders of magnitude lower than the first harmonic pressure amplitude.

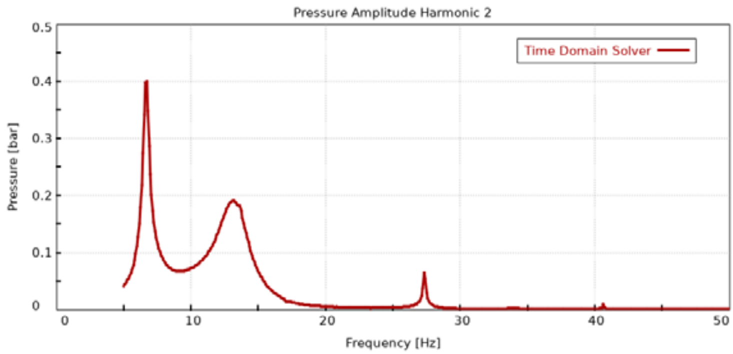


Figure 6: The second harmonic pressure amplitude at the end of the dead-end side branch as function of the flow oscillation frequency. These results have been obtained with the time domain solution method (method of characteristics). When using the harmonic solution method, the second harmonic pressure amplitude is identical to zero.

The coupling between harmonic components tends to be weak in general, but there might be situations in which this coupling leads to a more pronounced excitation of higher-order acoustic modes in a piping system. This effect will be captured by the time-domain solution method but not by the linear or modified harmonic solution method.

4 ACCURACY AND PERFORMANCE ASSESSMENT

To assess the accuracy and performance of the modified harmonic solution method, it has been used to calculate the pressure and flow rate oscillations in a collection piping models involving reciprocating pumps and compressors. Four of those models – labelled A, B, C and D – will be considered in this section. Here is a brief description of each model.

- **Model A** represents the discharge piping system of a triplex (three-cylinder) pump. A gas-filled dampener is located near the discharge side of the pump to reduce the pressure oscillations in the piping system.
- **Model B** represents an inter stage compressor system comprising six multi-chamber dampeners, three scrubbers and an assortment of process piping.
- **Model C** represents another inter stage compressor system comprising three multi-chamber dampeners, a heat exchanger and two knock-out drums for collecting any water condensate from the gas.
- **Model D** represents a single-stage compressor system comprising two double-acting compressors, eight spherical dampeners and two heat exchangers. This model contains some side branches connected to pressure safety valves. As these valves are closed during normal operation of the system, the steady state flow rate in these branches is zero.

These models, except for Model C, represent real-world pump and compressor systems. Model C (see Figure 7) has been created for training purposes and is therefore less detailed than the other models. Nonetheless, it does contain similar components and has a similar layout as a typical inter stage compressor system.

The flow excitation sources in these models are the discharge flow rates associated with the pumps and compressors. These flow rates are not purely sinusoidal as in the dead-end branch model. Instead, they follow a kind of saw-tooth pattern (see Figure 8) and therefore excite multiple frequencies simultaneously.

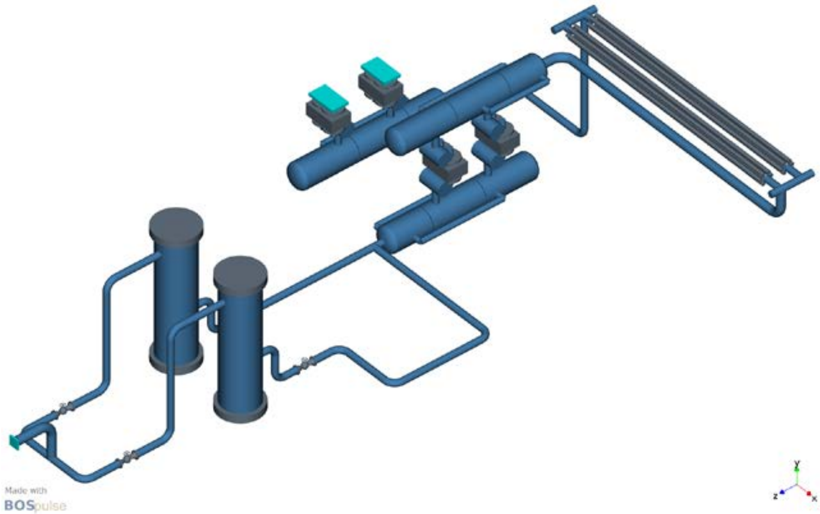


Figure 7: Model C, an inter stage compressor system. The other models feature similar components.

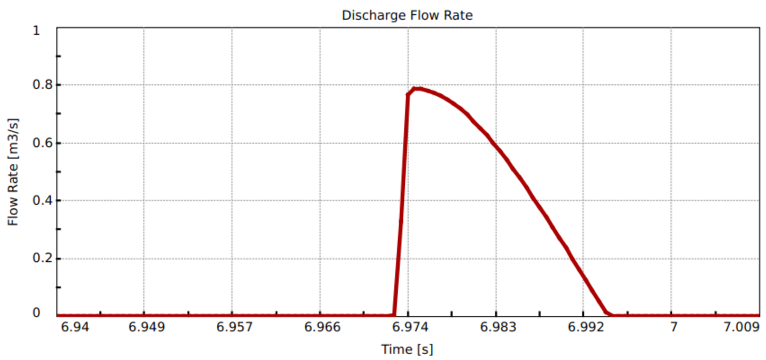
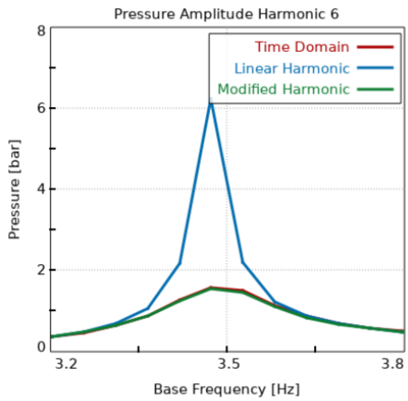
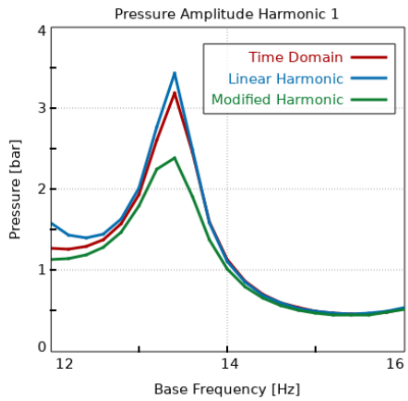


Figure 8: The flow rate that is discharged from one of the compressor cylinders in Model B during one time period (one rotation of the crank shaft).

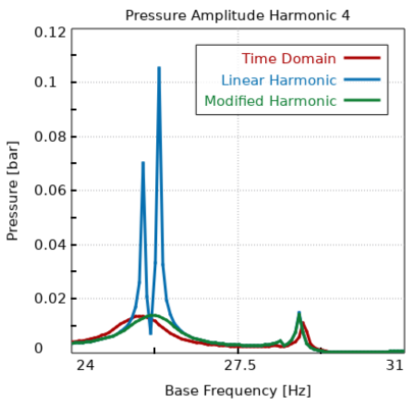
Figure 9 compares the pressure oscillations calculated with the time-domain method (method of characteristics), the linear harmonic solution method and the modified solution method for all four models. To be precise, the figure shows the pressure amplitudes as function of the base frequency with which the reciprocating equipment is operating. The pressure amplitudes are shown for the harmonic components and at locations for which the highest amplitudes are obtained. Note that these locations are limited to regular piping section as those are typically of interest.



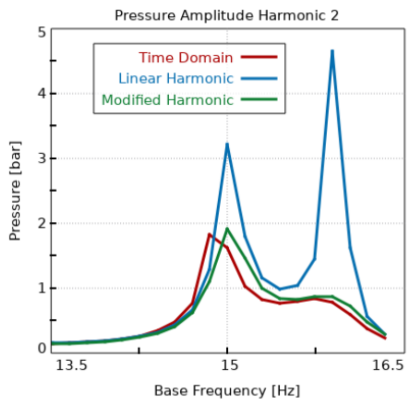
Model A



Model B



Model C



Model D

Figure 9: The pressure amplitudes as calculated with the time-domain method, the linear harmonic solution method, and the modified harmonic solution method as function of the base frequency with which the reciprocating equipment is operating.

Except for Model B, the modified harmonic solution method yields results that are close to the results obtained with the time-domain method. The original, linear solution method, on the other hand, yields results that are significantly different from the results obtained with the time-domain method.

Figure 10 shows the pressure amplitude associated with the first harmonic component at a location in a dampener that is part of Model B. As can be observed, the linear harmonic solution method yields results that differ significantly from the results obtained with the other two solution methods at this location in the model. Even though the pressure oscillations within a dampener tend to be less critical than the pressure oscillations in regular piping, they are relevant when calculating the shaking forces acting on the dampeners. These forces will be significantly overestimated when using the linear harmonic solution method.

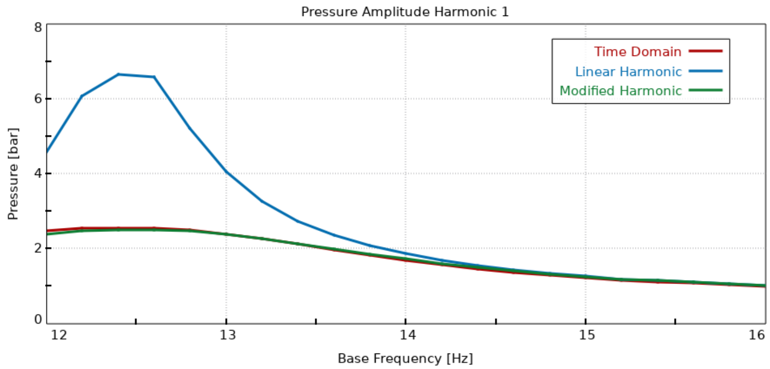


Figure 10: The pressure amplitude within a dampener located in Model B.

The modified harmonic solution method requires longer calculation times than the original, linear solution method. The difference, however, is not much; see Table 2. Indeed, the modified harmonic solution method is at most about two times slower than the linear harmonic solution method for the four test cases. In spite of that, it is still one or two orders of magnitude faster than the transient solver that has been used to obtain the results in the time domain. This solver is based on the method of characteristics and applies various optimisations to reduce its execution time (7).

Table 2: The relative execution times of the modified harmonic solution method and the time-domain method with respect to the execution times of the linear harmonic solution method for the four models.

	Modified Harmonic	Time Domain
Model A	1.75	217
Model B	2.16	74.6
Model C	1.04	7.54
Model D	1.18	81.0

5 CONCLUSIONS

The linear harmonic solution method can be modified in a relatively straightforward way to increase its accuracy considerably, especially when the steady state flow rate tends to zero. One modification concerns the application of an effective friction mass flow rate in order to approximate the non-linear friction term. The other modification concerns the use of an effective friction coefficient when calculating the steady state solution.

The two modifications make the harmonic solution method significantly more accurate. Indeed, the modified harmonic solution method tends to produce results that differ less than 10% from the results obtained with the time-domain solution method, whereas the original, linear solution method can produce pressure oscillations that are many times too large. This does not mean that the linear harmonic solution method has no merits, but it should be used with care.

The modified harmonic solution method involves a non-linear solution algorithm that requires more time to execute than the linear harmonic solution method. However, in practice only a few non-linear iterations are required so that the modified harmonic solution method is still one or two orders of magnitude faster than the time-domain solution method.

Although the modified harmonic solution method is more complex than the linear one, the additional complexity is relatively small. The reason is that at its core the modified harmonic solution algorithm essentially executes the linear solution method followed by an additional post-processing step.

Because of its large benefits the modified harmonic solution method has replaced the conventional time-domain method in a commercial pulsation analysis application.

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