Foam Drainage Analysis for a Tangential Cyclone

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Abstract

A qualitative study is performed of the foam drainage process in a tangential cyclone. Fundamental relations are derived that describe the drainage process in a cyclone. With these relations it is possible to capture the effects of the various medium, geometry and flow parameters on the efficiency of the drainage process and determine the drainage times for specific cyclone configurations.
Based on the derived relations estimations are made of the drainage times and efficiencies for a number of cyclone configurations. Furthermore recommendations are given to improve the drainage efficiency.

1. Introduction

The study of foam has a long history and can be found in many disciplines. Many of these studies focus on the mechanics of foam in a steady state condition. Under normal gravity conditions however foam is very seldom in a steady state. Foam is a structure built from gas cells joined by thin liquid films. The fluid in the film is subjected to gravity forces and will therefore drain from the film. Surface tension forces as well as viscosity will counteract this draining process. A large number of studies were devoted to address the dynamical and mechanical properties of foam.
However in spite of these numerous studies and investigations the detailed dynamics of foam drainage has only been studied very recently. The foam models used in most of these studies are basically one-dimensional representations of the foam. Foam is a dispersion of gas bubbles in a liquid. Most foam are structured two-phase compressible fluids with a systematic hexagonal texture as shown in the figure below.
The foam structure is built from liquid films, lamellas and so called Plateau borders. In the cross section of regularly structured foam a Plateau border is at the interface between three lamellas joining at an angle of 120°. In a three-dimensional foam structure four Plateau borders meet at an angle of 109°.
The process of foam drainage describes the depletion of the liquid levels from an initially liquid filled foam. In the drainage process the lamellas and films are thinning and coalescence of the various gas cells occurs. The amount of liquid that is drained per unit time depends on the stability of the foam and is governed by the interplay of i) acceleration forces, ii) capillary effects, iii) surface tension, iv) viscous forces, v) electric double-layer repulsion and vi) dispersion force attraction.

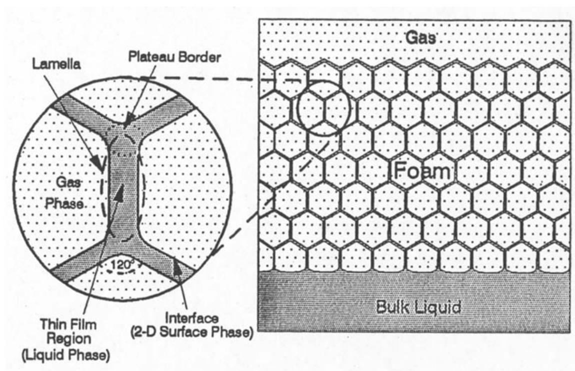

Figure 1: Representation of a foam.

2. The Drainage Equation

Foam is a disordered material and therefore we are interested in an average equation describing the decreasing liquid in foam. From studies performed by Koehler et al. (1998) such an equation is given for a one-dimensional foam.
In this equation the drainage process is described by the typical cross sectional area $A(z,t)$ of a Plateau border channel in a volume element consisting of many bubbles. The evolution of initially uniform foam of length L is considered. The 3D foam drainage equation can then be written as:

$$\frac{\partial A}{\partial t} + \frac{\rho g}{\eta}\frac{\partial A^2}{\partial z} - \frac{\gamma \delta^2}{2\eta}\boldsymbol{\nabla} \bullet (A^{1/2}\boldsymbol{\nabla} A) = \mathbf{0} \qquad (1)$$

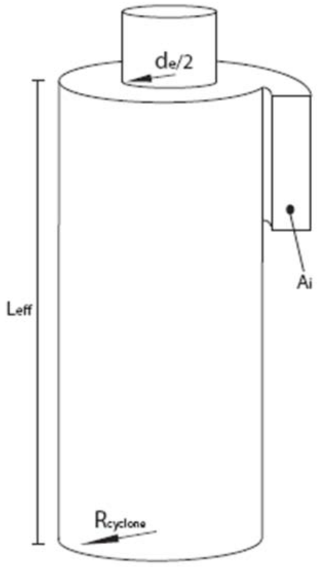

Figure 2: Parametric representation of a tangential inlet cyclone.

In one dimension the equation reduces to:

$$\frac{\partial A}{\partial t} + \frac{\rho g}{\eta}\frac{\partial A^2}{\partial z} - \frac{\gamma\delta^2}{2\eta}\frac{\partial}{\partial z}(A^{1/2}\frac{\partial A}{\partial z}) = \mathbf{0} \qquad (2)$$

The one-dimensional foam drainage equation that was derived in (Koehler et al., 1998) can be used to obtain a simple foam drainage model for a tangential inlet cyclone. A typical tangential cyclone with an uniform barrel of height L_{eff} is shown in figure 2.

The assumption is made that when the foam enters the cyclone it instantaneously distributed over a sheet Ω. The liquid is drained from this sheet along the radial direction z only; no variation of drainage occurs along the height of the sheet. As the foam sheet moves along the tangential direction of the cyclone fluid is drained from the foam until time t_{res} is reached, from when the sheet ceases to exist. A representation of this model is shown below, where different time values are plotted rather than sheet positions in the barrel due to the spiral motion of the sheet.

For this model it is possible to apply the foam drainage equation that was discussed in the previous section.

To do so the radial coordinate z is defined as the distance along the radius of the cyclone barrel. Furthermore, the drainage equation is modified to account for the variation in centrifugal acceleration along the radial coordinate. By doing so the following drainage equation is obtained:

$$\frac{\partial A}{\partial t} + \frac{\rho}{\eta}\frac{\partial g \cdot A^2}{\partial z} - \frac{\gamma\delta^2}{2\eta}\frac{\partial}{\partial z}(A^{1/2}\frac{\partial A}{\partial z}) = \mathbf{0} \qquad (3)$$

$$g(z) = \frac{V_\theta^2}{z} \qquad (4)$$

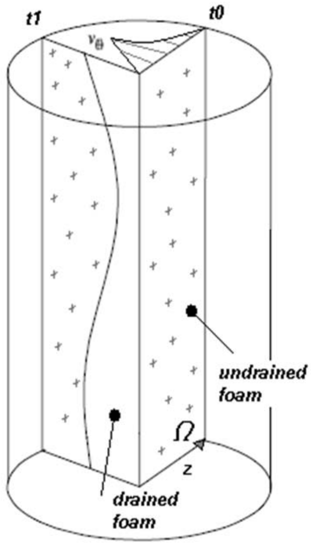

Figure 3: Representation of the foam drainage model.

The tangential velocity of this model is obtained from the following expression:

$$V_\theta = \frac{\Gamma}{2\pi}\left(\frac{z^2}{z^2 + r_c^2}\right)\left(\frac{1}{z}\right) \qquad (5)$$

When the expression for the tangential velocity is substituted in (4), the following modified drainage equation is obtained:

$$\frac{\partial A}{\partial t} + \frac{\rho}{\eta}\frac{\Gamma}{2\pi}\frac{\partial}{\partial z}\left(\frac{A^2}{z}\cdot\left(\frac{z}{z^2+r^2}\right)^2\right) - \frac{\gamma\delta^2}{2\eta}\frac{\partial}{\partial z}(A^{1/2}\frac{\partial A}{\partial z}) = \mathbf{0} \qquad (6)$$

3. Drainage Process in a Cyclone

Since capillary forces are much smaller in comparison to the centrifugal forces in a cyclone they may be neglected. From 7 the following modified drainage equation can be derived:

$$\begin{aligned} \frac{\partial A}{\partial t} + \frac{\rho g_0}{\eta}\frac{\partial}{\partial z}\left(\frac{A^2}{z}\cdot\left(\frac{z}{z^2+r^2}\right)^2\right) &= \qquad (7)\\ \frac{\partial A}{\partial t} + k_2\frac{\partial}{\partial z}\left(\frac{A^2}{z}\cdot\left(\frac{z}{z^2+r^2}\right)^2\right) &= 0 \end{aligned}$$

Where g_0 is defined to be:

$$g_0 = \left(\frac{\Gamma}{2\pi}\right)^2 \qquad (8)$$

This first order partial differential equation may be solved analytically and the following solution is obtained:

$$A(z,t) = \begin{cases} \frac{1}{5}\frac{6z^2r_c^2+5r_c^4+z^4}{k_2t} & \text{, z} \leq \text{z}_1, t < t^* \\ \delta R^2 & \text{, t} \leq t^* \\ \frac{1}{5}\frac{6z^2r_c^2+5r_c^4+z^4}{k_2t} & \text{, z} > \text{z}_1, t \geq t^* \end{cases}$$

The above equation is applicable for most cyclones and in particular when the centrifugal forces are large in comparison to the capillary forces foam drainage may be sufficiently well approximated by these solutions.

4. Drainage Efficiency and Scaling

From the solution obtained in section 3 it is relatively easy to obtain the efficiency of the drainage process in a cyclone. Furthermore, useful expressions may be derived that can be used to scale the drainage solution obtained for a particular cyclone and flow medium, to acquire a new solution for a different cyclone and medium configuration.
The overall efficiency of drainage at a particular time t can be written as:

$$\Psi_t = 1 - \frac{\int_0^{R_{cyclone}} A(z,t)dz}{\delta R^2 R_{cyclone}}, \qquad (9)$$

where Ψ is defined to be the efficiency.
When the solution of the drainage equation is substituted in the above equation the drainage time at a specified efficiency is defined to be:

$$[l]t = \frac{1}{25}\frac{(10R_{cyclone}^2 r_c^2 + 25r_c^4)\eta R_{cyclone}^2}{\rho V_i^2 (R_{cyclone}^2 + r_c^2)^2 \delta R^2 (1-\Psi)}, \qquad (10)$$
$$(\Psi > 0.5)$$

To obtain an useful scaling relation, all constant terms and factors in the above relation may be omitted. If it is assumed that the ratio between the inlet pipe r_c and the radius of the barrel $R_{cyclone}$ will not be varied in the scaling process, the following scaling relation is obtained:

$$t \propto \frac{\eta R_{cyclone}^2 \Psi}{\rho V_i^2 \delta R^2}, (\Psi > 0.5). \qquad (11)$$

5. Results

With the derived relations the drainage time for a number of cyclone configurations are determined. An example for one cyclone is shown in figure 4. Where the percentage of drained fluid along the radius of the barrel is shown for a number of time steps.
From the determined relations recommendations have been given to improve the performance of various cyclones.

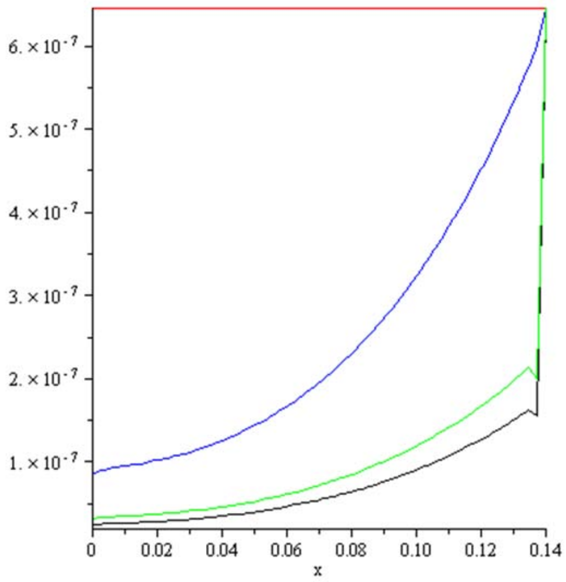

Figure 4: Percentage of the drained fluid along the radius of the barrel.

6. Conclusions

This article describes the qualitative study of foam drainage in a tangential cyclone. A number of fundamental analytical relations are derived to determine the drainage efficiency of the cyclone.
Furthermore these relations are applied to determine the efficiency for a number of cyclone configurations.

7. Bibliography

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